Homework #2

STAT675 (Rice University) or GS010103 (GSBS)

(a) Prove Theorem 3 from Slides of Week 4 (Feb 4th)

Consider the linear model where both β and τ are unknown. Then prove that the Jeffrey's joint prior is given by

$$\pi(\beta|\tau) \propto \tau(X'X)^{p/2-1}$$

(b) Prove Theorem 5 from Slides of Week 4 (Feb 4th)

Consider the linear model where both β and τ are unknown.

$$\begin{aligned}
\pi(\beta|\tau) &\propto \text{Normal}_p(\mu_0, \tau^{-1}\Sigma_0) \\
\pi(\tau) &\propto \text{Gamma}(\delta_0/2, \gamma_0/2)
\end{aligned}$$

Then prove that β is $\beta|y \sim \mathrm{S_p(n + \delta_0, \beta^*, \tilde{s}^2(X'X + \Sigma_0^{-1})^{-1})}$ where S_p is the p-variate Students t-distribution with $(n + \delta_0)$ degrees of freedom and the posterior of τ is $\mathrm{Gamma\{(n + \delta_0)/2, \bar{s}^2(n + \delta_0)/2\}}$, where

$$\begin{aligned}
\beta^* &= \Lambda\mu_0 + (I - \Lambda)\hat{\beta}, \\
\Lambda &= (X'X + \Sigma_0^{-1})^{-1}\Sigma_0^{-1}, \\
\hat{\beta} &= (X'X)^{-1}X'Y. \\
\bar{s}^2 &= (n + \delta_0)^{-1}[Y'(I - M)Y + (\hat{\beta} - \mu_0)'(\Lambda'X'X)(\hat{\beta} - \mu_0) + \gamma_0], \\
M &= X'(X'X)^{-1}X.
\end{aligned}$$