Homework 2

GCSR, problems 2.5, 2.9c (use Jeffreys' prior), 2.11. And the following problems:

1. This example shows how in a very simple setup a (frequentist) hypothesis testing rule violates the likelihood principle. (Berger & Wolpert 1984)

Assume we have an unknown parameter θ, with $\theta \in \{0, 1\}$. Consider the following two experiments.

	$x_1 :$	1	2	3
E1:	$p(x_1\|\theta = 0) =$	0.90	0.05	0.05
	$p(x_1\|\theta = 1) =$	0.09	0.055	0.855
	$x_2 :$	1	2	3
E2:	$p(x_2\|\theta = 0) =$	0.26	0.73	0.01
	$p(x_2\|\theta = 1) =$	0.026	0.803	0.171

For the hypothesis $H_0 : \theta = 0$, define the following test rule δ: Reject H_0 if $x_i = 2$ or $x_i = 3$ *and* report the type-I and type-II error rates.

(a) Under experiment E1, find the type-I and type-II error probabilities of hypothesis test δ.
(b) Same under experiment E2.
(c) Show that δ violates the likelihood principle.

2 (Robert 1994) While working on the audience share of a TV series, $0 \le \theta \le 1$ representing the part of the TV audience, an investigator found nine viewers and three nonviewers. If no additional information is available on the experiment, two probability models at least can be proposed:
E1: the investigaator questioned 12 persons, thus observed $x \sim Bin(12, \theta)$ with $x = 9$.
E2: the investigator questioned persons until he/she obtained 9 viewers, thus observed
$x \sim Neg-Binom(\alpha = 9, \theta)$ with $x = 3$ (Note: we use here the "alternative" parameterization of the negative binomial with α = number of successes, $\theta = Pr(success)$. See p483 GCSR).

Consider the hypothesis $H_0 : \theta = 1/2$ versus $H_1 : \theta > 1/2$. We use the following rule δ: Under E1, reject H_0 if $x > x^*$, where x^* is the smallest x^* s.t. $Pr(x > x^*|H_0) \ge \alpha$ (i.e. x^* defines the critical region for a size α test). Use $\alpha = 0.05$.
Under E2, reject H_0 if $x < x^{**}$, where x^{**} is the largest x^{**} s.t. $Pr(x < x^{**}|H_0) \ge \alpha$ (i.e. x^{**} defines the critical region for a size α test). Use $\alpha = 0.05$.

(a) Show that δ is the UMP ("uniformly most powerful") test at size $\alpha = P(x > x^*|\theta = 1/2)$ under E1. See, for example, DeGroot p466-473 for a definition of UMP tests.[1]
(b) Show that δ is the UMP ("uniformly most powerful") test at size $\alpha = P(x < x^{**}|\theta = 1/2)$ under E2.[2]
(c) Deduce that frequentist theory of tests is not compatible with the Likelihood principle.[3]

[1] If you don't know about UMP tests from elsewhere, skip parts (a) and (b). Please try though and make a note

3 Show that the stopping rule principle (as defined in class) is a consequence of the likelihood principle.

4 (Robert 1994) Consider $x_i \sim Exp(\lambda)$, $i = 1, \ldots, n$. The data is *censored* in the sense that there exist n random variables $y_1, \ldots, y_n$ distributed according to $p_Y(y)$, independent of λ, and $z_i = \min(x_i, y_i)$, together with an indicator $w_i = 1(z_i = y_i)$ are actually observed.
Show that, according to the Likelihood Principle, the inference on λ should not depend on p_Y.