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Outline

- Big Data
- BIAS and Big Data Integration
- Image-on-Scalar Models
- Image-on-Genetic Association Models
- Prediction Models

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What is 'Big Data'?

5V=Volume, Velocity, Variety, Value, and Veracity

The size of big data is beyond the ability of commonly used software tools to capture, manage, and process within a tolerable elapsed time.

- Alzheimer's Disease Neuroimaging Initiative (US\$134 millions)
- Philadelphia Neurodevelopmental Cohort (PNC)
- Human Connectome Project (HCP)

Big Data in Boxes



How to promote statistics in 'Big Data' industry?

- Closely collaborate with people who are collecting 'Big Data'
- Work as a team to develop new methods, packages, and textbooks with nice case studies
- Organize more workshops and short courses
- Train next-generation statisticians: training grants and new courses a data scientist; an excellent programmer; an applied mathematician

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- Start with a few big data bases
- Start with a few methodological and clinical projects
- Develop a package with a set of good computational and statistical tools to efficiently extract important information from large Big data

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BIAS: Biostatistics and Imaging AnalysiS and Big Data Integration





BIAS: Biostatistics and Imaging Analysis Lab



Man Power Computer Power Programming Power Statistics Mathematics

http://www.bios.unc.edu/research/bias/





Human Brain Project

aims to simulate the complete human brain on Supercomputers to better understand how it functions BR



BRAIN Funding Opportunities

The Brain Research through

Advancing Innovative Neurotechnologies or BRAIN, aims to reconstruct the activity of every single neuron as they fire simultaneously in different brain circuits, or perhaps even whole brains.









Big Neuroimaging Data

NIH normal brain development 1000 Functional Connectome Project Alzheimer's Disease Neuroimaging Initiative National Database for Autism Research (NDAR) Human Connectome Project Philadelphia Neurodevelopmental Cohort Genome superstruct Project









www.guysandstthomas.nhs.uk/.../T/Twins400.jpg



Complex Study Design

cross-sectional studies; clustered studies including longitudinal and twin/familial studies;







Neuroimaging Applications



- Variety of acquisitions
- Measurement basics
- Limitations & artefacts
- Analysis principles
- Acquisition tips









Complex Data Structure

Multivariate Imaging Measures Smooth Functional Imaging Measures Whole-brain Imaging Measures 4D-Time Series Imaging Measures





Big Data Integration





Big Data Integration





Image-on-Scalar Models



Big Data Integration





The NIMH Strategic Plan

- Strategic Objective 1: Promote Discovery in the Brain and Behavioral Sciences to Fuel Research on the Causes of Mental Disorders
- Identifying and validating high sensitivity and specificity biomarkers that define valid subtypes of the major mental illnesses.
- Strategic Objective 2: Chart Mental Illness Trajectories to Determine When, Where, and How to Intervene
- Conducting longitudinal studies that track changes in behavior with brain structure, connectivity, and function, in order to characterize the progression from primary changes to subsequent clinical presentation, and to identify predictors of divergence from the typical trajectory.

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Smoothed Functional Data



Covariates (e.g., age, gender, diagnostic)

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Case 1: DTI Fiber Tract Data

Data



• Diffusion properties (e.g., FA, RA)

$$Y_i(s_j) = (y_{i,1}(s_j), \cdots, y_{i,m}(s_j))^T$$

• Grids $\{s_1, \cdots, s_{n_G}\}$

• Covariates (e.g., age, gender, diagnostic) x_1, \cdots, x_n

Longitudinal Tract Data

Longitudinal Data

Spatial-temporal Process

 $t \wedge y_i(s,t_3) \\ y_i(s,t_2) \\ y_i(s,t_1)$ Functional Mixed Effect Models

$$y_i(s,t) = x_i(t)^T B(s) + z_i(t)^T \xi_i(s) + \eta_i(s,t) + \varepsilon_i(s,t)$$

Objectives: Dynamic functional effects of covariates of interest on functional response.

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Ex 1: Longitudinal Tract Data

| Gender: Male/Female | 83/54 |
|----------------------------------|---------------------|
| Gestational age at birth (weeks) | 38.67 ± 1.74 |
| Age at scan 1 (days) | 297.89 ± 13.90 |
| Age at scan 2 (days) | 655.34 ± 24.00 |
| Age at scan 3 (days) | 1021.70 ± 28.26 |
| Number of Gradient directions | |
| dir6/dir42 at scan 1 | 80/24 |
| dir6/dir42 at scan 2 | 59/44 |
| dir6/dir42 at scan 3 | 42/49 |
| | |

| Available scans | N |
|--------------------------------|----|
| Neonate scan only | 1 |
| 1 year scan only | 2 |
| 2 year scan only | 3 |
| Neonate + 1 year scan | 43 |
| Neonate + 2 year scan | 30 |
| 1 year + 2 year scan | 28 |
| Neonate + 1 year + 2 year scan | 30 |

DTImaging parameters:

- TR/TE = 5200/73 ms
- Slice thickness = 2mm
- In-plane resolution = 2x2 mm²
- b = 1000 s/mm^2
- One reference scan b = 0 s/mm²
- Repeated 5 times when 6 gradient directions applied.

Ex 1: Longitudinal Tract Data

Ex 1: Longitudinal Tract Data

Neuroimaging Data with Discontinuity

Noisy Piecewise Smooth Function with Unknown Jumps and Edges

Covariates (e.g., age, gender, diagnostic, stimulus)

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Case 2: Piecewise Smooth Data

Mathematics.

Noisy Piecewise Smooth Functions with Unknown Jumps and Edges

Image is the point or set of points in the range corresponding to a designated point in the domain of a given function.
 ▲ Ω is a compact set. x̃ ∈ Ω ⊆ R^k

 $\longrightarrow f(\tilde{x}) \in M \subseteq R^m \qquad f: \Omega \to M \subseteq R^m$

M2: Spatial Varying Coefficient Model

Decomposition:

$$y_{i}(d) = f(x_{i}, B(d) + \eta_{i}(d)) + \varepsilon_{i}(d), d \in D$$

$$\xrightarrow{\text{Piecewise Smooth}} Short-range Correlation} Short-range Correlation} Short-range Correlation} Short-range Correlation} \varepsilon_{ij}(\bullet) \sim SP(0, \Sigma_{\eta})$$

$$\varepsilon_{ij}(\bullet) \sim SP(0, \Sigma_{\eta})$$

Covariance operator:

$$\Sigma_{y}(d,d') = \Sigma_{\eta}(d,d') + \Sigma_{\varepsilon}(d,d)$$

Li, Zhu, Shen, Lin, Gilmore, and Ibrahim (2011). JRSSB. Zhu, Fan, and Kong (2014) JASA

Spatial Varying Coefficient Model

Cartoon Model

$$B_k(d)$$

- **Disjoint Partition** $D = \bigcup_{l=1}^{L} D_l$ and $D_l \cap D_{l'} = \phi$
- Piecewise Smoothness: Lipschitz condition
- Smoothed Boundary
- Local Patch
- Degree of Jumps

Kernel-based Smoothing Methods

$$y = f + \varepsilon;$$
 ε uncorrelated, mean=0, var= σ^2

Estimate f_i as a weighted average of the noisy pixels:

$$\widehat{f}_i = \sum_j w_{i,j} y_j$$

Arias-Casto, Salmon, Willett (2011)

- Local constant/linear
- Yaroslavsky/Bilateral Filter
- Nonlocal Means

• PS

Kernel-based Smoothing Methods

Propogation-Seperation Method J. Polzehl and V. Spokoiny, (2000,2005)

Noisy image sigma=0.4

Reconstruction local constant PS

Reconstruction local quadratic PS

nonadaptive kernel smoothing

Features

Increasing Bandwidth

- Adaptive Weights
- Adaptive Estimates

Simulation

True Image

SVCM

Initial Estimate in SVCM

Estimate with LF and r=2

Estimate with LF and r=1

Estimate with LF and r=0

Simulation

EX2: ADNI PET Data

- Data were obtained from the Alzheimer's Disease Neuroimaging Initiative (ADNI) database.
- We consider PET scans obtained at baseline, 6 months, and 12 months.
- Subjects are classified as having mild cognitive impairment (MCI), as AD patients, or as Normal Controls (NC).

| Diagnostic status | age (years) | N male | N female |
|-------------------|---------------|--------|----------|
| AD | 75.9 ± 6.9 | 34 | 17 |
| MCI | 76.3 ± 7.3 | 33 | 25 |
| NC | $77.0\pm$ 4.2 | 30 | 20 |

- We randomly chose 80 subjects for the training set to develop the prediction model.
- We predicted the PET scans at month 12, based on the baseline and 6-month scans for 79 subjects in the test set.
- We used gender, diagnostic status (MCI, AD, NC), and age (55-90 years) as covariates for the semi-parametric model.

Hyun, J.W., Li, Y. M., J. H. Gilmore, Z. Lu, M. Styner, H. Zhu (2014) SGPP. NeuroImage

EX2: ADNI PET Data

Figure : Observed (upper panel) and predicted (bottom panel) PET images at month 12 for (a) an AD patient, (b) an MCI subject, and (c) a NC subject. One selected slice is shown.

EX2: ADNI PET Data

Figure : rtMSPE maps for prediction of ADNI PET images at month 12 for 79 test subjects. Selected slices are shown for (a) Semi-parametric model; (b) Semi-parametric model+FPCA; (c) Semi-parametric model+FPCA+Spatial-temporal model.

| Semi-parametric model | 0.0692 |
|---|--------|
| Semi-parametric mode+FPCA | 0.0550 |
| Semi-parametric model+FPCA+Spatial-temporal model | 0.0354 |

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Image-on-Genetic Association Models

The NIMH Strategic Plan

- Strategic Objective 1: Promote Discovery in the Brain and Behavioral Sciences to Fuel Research on the Causes of Mental Disorders
- Identify the genetic and environmental factors associated with mental disorders.
- **Strategic Objective 2: Chart Mental Illness Trajectories to Determine** When, Where, and How to Intervene
- When identifying behavioral, neural, and/or genetic markers along the trajectory of illness, design the studies to consider variation in relation to age, sex, gender, race, ethnicity, and other important socio-demographic factors.

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Big Data Integration

Statistical Methods

Hibar, et al. HBM 2012

Data Structure

M3: High Dimensional Regression Model

Data $\{(Y_i, X_i) : i = 1, \dots, n\}$

$$Y_i = \{y_i(v) : v \in V\}$$

$$\{X_i(g):g\in G_0\}$$

Sparse Projection Regression Model

• Let $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_k]$, then a projection regression model is given by:

$$\mathbf{W}^{\mathsf{T}} y_i = (\mathbf{B} \mathbf{W})^{\mathsf{T}} \mathbf{x}_i + \mathbf{W}^{\mathsf{T}} \mathbf{e}_i = \beta_{\mathbf{w}}^{\mathsf{T}} \mathbf{x}_i + \varepsilon_i$$

• Hypothesis problem reduces to:

$$H_{0W} : \mathbf{C}\beta_{\mathbf{w}} = \mathbf{b}_{0} \quad v.s. \quad H_{1W} : \mathbf{C}\beta_{\mathbf{w}} \neq \mathbf{b}_{0}$$

where $\mathbf{C}\beta_{\mathbf{w}} = \mathbf{CBW}$ and $\mathbf{b}_{0} = \mathbf{B}_{0}\mathbf{W}$

• How to determine an 'optimal' W?

Sun, Zhu, Liu, and Ibrahim (2014) JASA

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Sparse Projection Regression Model

 We show that this is achieved by optimizing the following generalized heritability ratio (GHR):

$$\mathsf{GHR}(\mathbf{w}; \mathbf{C}) = \frac{\mathbf{w}^T (\tilde{\mathbf{B}}_1 - \mathbf{B}_0)^T S_{\tilde{X}_1} (\tilde{\mathbf{B}}_1 - \mathbf{B}_0) \mathbf{w}}{\mathbf{w}^T \Sigma_R \mathbf{w}} = \frac{\mathbf{w}^T \Sigma_C \mathbf{w}}{\mathbf{w}^T \Sigma_R \mathbf{w}}$$

- High Dimensional Setting
- noise accumulation
 - ill-conditioned sample covariance estimator: $\hat{\Sigma}_R$
- Sparse Projection Regression Model is proposed as following:

$$\operatorname{argmax} \{ \frac{\mathbf{w}^T \hat{\boldsymbol{\Sigma}}_C \mathbf{w}}{\mathbf{w}^T \tilde{\boldsymbol{\Sigma}}_R \mathbf{w}} \} \quad \text{s.t.} \ ||\mathbf{w}||_1 \le t$$

Sparse and Low-rank Representation

Regularization Methods

- Lasso 1, 2, 3,
- SCAD, MCP,

Shen, Shen, and Zhu (201?)

$$\widehat{\theta} \in \arg\min_{\theta} \frac{1}{n} \sum_{i=1}^{n} (y_i - x_i^T \theta)^2 + \lambda_n \sum_{j=1}^{p} |\theta_j|$$

Factor Model

M4: Voxel-wise GWAS

M4: Voxel-wise GWAS

Fast Sure-Independence Screening Procedure

EX3: 93 ROI-GWAS

EX4: Whole Brain-GWAS

Prediction Models

Alzheimers Disease Big Data DREAM Challenge 1

Its goal is to apply an open science approach to rapidly identify **accurate predictive AD biomarkers** that can be used by the scientific, industrial and regulatory communities to improve AD diagnosis and treatment.

- **Sub 1:** Predict the change in cognitive scores 24 months after initial assessment.
- **Sub 2:** Predict the set of cognitively normal individuals whose biomarkers are suggestive of amyloid perturbation.
- **Sub 3:** Classify individuals into diagnostic groups using MR imaging.

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Big Data Integration

HRM versus FRM

Data
$$\{(y_i, X_i) : i = 1, \dots, n\}$$
 $X_i = \{X_i(d) : d \in D\}$
 $y_i = \langle X_i, \theta \rangle + \varepsilon_i$

Strategy 1: Discrete Approach (High-dimension Regression Model (HRM))

Strategy 2: Functional Regression Model (FRM)

$$y_i = \theta_0 + \int_D \theta(d) X_i(d) m(d) + \varepsilon_i$$

High-dimension Regression Model

Approach 1: Regularization Methods

Key Conditions:

- Sparsity of S
- Restricted null-space property for design matrix X

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High-dimension Regression Model

 $n \times p$

n

CP decomposition

Tucker decomposition

 S^c

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Scalar-on-Image Models

Simulations

Key Conditions:

- Tensor Approximation B
- Restricted space property for X and B

Scalar-on-Image Models

Strategy 2: Functional Approach

$$y_{i} = \theta_{0} + \int_{D} \theta(d) X_{i}(d) m(d) + \varepsilon_{i}$$
$$\theta(d) = \sum_{k=1}^{\infty} \theta_{k} \psi_{k}(d)$$
$$y_{i} = \theta_{0} + \sum_{k=1}^{\infty} \theta_{k} \int_{D} \psi_{k}(d) X_{i}(d) m(d) + \varepsilon_{i}$$

Basis Methods: fixed and data-driven basis functions

Key Conditions

Key Conditions: an excellent set of basis functions

- Sparsity of basis representation $\{\theta_k : k = 1, \dots\}$
- Decay rate of spectral of C or $K^{1/2}CK^{1/2}$

$$\theta(d) \approx \sum_{k=1}^{K} \theta_k \psi_k(d) \qquad K << n$$

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Extensions

- M5: Functional linear Cox regression models
- M6: Generalized scalar-to-image regression models
- M7: Multiscale Functional Linear models

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M5: Functional Linear Cox Regression Model

Data
$$\{(y_i, X_i) : i = 1, \dots, n\}$$
 $X_i = \{X_i(d) : d \in D\}$

 $y_i = \min(T_i, C_i)$ T_i : failure time; C_i : censored time Model

$$interpretation h(t) = f(t) / S(t) = h_0(t) \exp(z_i^T \gamma + \int_S X_i(s) \beta(s) ds)$$

$$X_i(s) = \mu(s) + \sum_{j=1}^{\infty} \xi_{ij} \phi_j(s) + \varepsilon_i(s)$$

- Consistency
- Asymptotic distribution of score test

M5: Functional Linear Cox Regression Model

Mild Cognitive Impairment subjects

Interested in predicting the timing of an MCI patient that converts to AD by integrating the imaging data, the clinical variables, and genetic covariates.

Full Model:AUC=0.96Partial Model:AUC=0.82

M6: Generalized scalar-to-image regression models

Data
$$\{(y_i, X_i) : i = 1, \dots, n\}$$
 $X_i = \{X_i(d) : d \in D\}$
Model $y \sim$ exponential family (μ, ϕ)
 $g(\mu) = \theta_0^T Z + \langle X, \beta_0 \rangle$
Total Variation
Estimation: $\sum_{i=1}^n \ell(y_i; \mu(X_i; \gamma, \beta(\bullet))) + \lambda \parallel \beta \parallel_{TV}$

Non-asymptotic Error Bound:

$$\mathcal{R}_{2n} = \left\{ \mathbb{E}^* \left(\left\langle X^{(n+1)}, \hat{\beta} - \beta_0 \right\rangle \right)^2 \right\}^{1/2},$$

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M6: Generalized scalar-to-image regression models

Triangle

T-shape

checkerboard

M7: Multiscale Functional Linear models

Data
$$\{(y_i, X_i): i = 1, \dots, n\}$$
 $X_i = \{X_i(d): d \in D\}$

Models

(A1)
$$D = (\bigcup_{k=1}^{K} D_k) \bigcup D_0$$
 • Informative sets + Irrelevant set

$$(A2) \quad y \perp \{X(d) : d \in D_0\}$$

(A3)
$$y \sim p(\{X(d) : d \in D_1\}, \dots, \{X(d) : d \in D_K\})$$

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Simulation I: Classification

$$X_i(d) = \beta_0(d) + \beta_1(d)y_i + \varepsilon_i(d)$$

| Type I | Type II | Type III | | |
|----------------|--|---|--|--|
| <i>N</i> (0,4) | <u>Short-range</u> <u>correlation</u> | <u>Long-range</u> <u>correlation</u> | | |

Simulation I: Classification

Table 1: Misclassification rates for PCA and SWPCA under the different number of PCs.

| Noise | Number of PCs | PCA | SWPCA1 | SWPCA2 | SWPCA3 |
|----------|---------------|------|--------|--------|--------|
| Type I | 5 | 0.40 | 0.11 | 0.09 | 0.10 |
| | 7 | 0.40 | 0.13 | 0.11 | 0.10 |
| | 10 | 0.40 | 0.13 | 0.11 | 0.10 |
| Type II | 5 | 0.40 | 0.04 | 0.08 | 0.03 |
| | 7 | 0.39 | 0.03 | 0.09 | 0.04 |
| | 10 | 0.38 | 0.03 | 0.07 | 0.04 |
| Type III | 5 | 0.40 | 0.13 | 0.10 | 0.09 |
| | 7 | 0.41 | 0.13 | 0.10 | 0.10 |
| | 10 | 0.41 | 0.13 | 0.10 | 0.10 |

Simulation I: Classification

| Noise | sLDA | sPLS | SLR | SVM | ROAD | PCA | SWPCA |
|----------|------|------|------|------|------|------|-------|
| Type I | 0.28 | 0.43 | 0.45 | 0.38 | 0.36 | 0.36 | 0.10 |
| Type II | 0.27 | 0.08 | 0.18 | 0.26 | 0.08 | 0.45 | 0.03 |
| Type III | 0.52 | 0.30 | 0.61 | 0.60 | 0.50 | 0.35 | 0.09 |

sLDA: sparse discriminant analysis sPLS: sparse partial least squares analysis SLR: sparse logistic regression SVM: support vector machine ROAD:

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EX5: ADNI-PET

AD

NC

ADNI

94 AD subjects and 104 NC subjects

Table 3: Results of Real Data: average misclassification rates.

| sLDA | sPLS | sLogistic | SVM | ROAD | PCA | SWPCA |
|-------|-------|-----------|-------|-------|-------|-------|
| 0.255 | 0.163 | 0.179 | 0.168 | 0.189 | 0.194 | 0.117 |

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Thank You!!

ASA: Statistics in Imaging Section

SAMSI 2013 Neuroimaging Data Analysis 2015-2016 Challenges in Computational Neuroscience July 27-31 Summer School August 17-21 Opening Workshop

- Shape Analysis
- Spike Train Analysis
- Big Data Integration
- Compressed Sensing
- Functional Data Analysis

