Homework 1

STAT675 & GS10103

1. Suppose R and G are independent random variables, both following gamma distributions given by

$$p(R|\theta_r, a) = \frac{\theta_r^a R^{a-1} \exp(-R\theta_r)}{\Gamma(a)} \text{ for } R > 0$$

and

$$p(G|\theta_g, a) = \frac{\theta_g^a G^{a-1} \exp(-G\theta_g)}{\Gamma(a)} \text{ for } G > 0.$$

Denote these two gamma distributions by $g(a, \theta_r)$ and $g(a, \theta_g)$, respectively. Suppose that θ_r and θ_g are independent and follow the same gamma distribution $g(a_0, \nu)$. Let $\rho = \theta_g/\theta_r$. Show that given a and a_0, the conditional posterior distribution

$$p(\rho|R, G, a, a_0) \propto \rho^{-(a+a_0+1)} \left\{ \frac{1}{\rho} + \frac{(G+\nu)}{(R+\nu)} \right\}^{-2(a+a_0)}$$

and

$$Mode(\rho|R, G, a, a_0) = \left(\frac{R+\nu}{G+\nu}\right)\left(\frac{a+a_0-1}{a+a_0+1}\right), E(\rho|R, G, a, a_0) = \left(\frac{R+\nu}{G+\nu}\right)\left(\frac{a+a_0}{a+a_0-1}\right).$$