Question 1:

With regard to Smyth's moderated t-statistics, we have the model and priors as,

$$\hat{\beta}_{gj} \sim N(\beta_{gj}, c_{gj}\sigma_g^2)$$

$$P(\beta_{gj} \neq 0) = p$$

$$\beta_{gj} \mid \beta_{gj} \neq 0 \sim N(0, c_{0j}\sigma_g^2)$$

$$s_g^2 \sim \frac{\sigma_g^2}{d_g}\chi_{d_g}^2$$

$$\sigma_g^2 \sim s_0^2(\chi_{d_0}^2/d_0)^{-1}$$

Please provide detailed derivation for the posterior variance estimators

$$\tilde{s}_g^2 = E(\sigma_g^2 \mid s_g^2).$$

Question 2:

Hu and Wright (2007) derived the limiting value of pFDR when δ is discrete in Section 3. If we have

$$\delta_i \sim \left\{ \begin{array}{ll} N(0, \sigma_0^2) & \text{with prob } \pi_0 \\ N(\delta', \sigma_1^2) & \text{with prob } 1 - \pi_0 \end{array} \right.$$

where δ', σ_0^2, and σ_1^2 are known values.

Please provide the derivation of the limiting value of FDR in this case. Please try your best to simply the formula for the limiting value.