Homework #2

STAT675 (Rice University) or GS010103 (GSBS)

(a) Prove Theorem 3 from Slides of Week 4 (Feb 4th)

Consider the linear model where both $\beta$ and $\tau$ are unknown. Then prove that the Jeffrey’s joint prior is given by

$$\pi(\beta|\tau) \propto \tau(X'X)^{p/2-1}$$

(b) Prove Theorem 5 from Slides of Week 4 (Feb 4th)

Consider the linear model where both $\beta$ and $\tau$ are unknown.

$$\pi(\beta|\tau) \propto \text{Normal}_p(\mu_0, \tau^{-1}\Sigma_0)$$

$$\pi(\tau) \propto \text{Gamma}(\delta_0/2, \gamma_0/2)$$

Then prove that $\beta|y \sim S_p(n + \delta_0, \beta^*, \bar{s}^2(X'X + \Sigma_0^{-1})^{-1})$ where $S_p$ is the p-variate Students t-distribution with $(n + \delta_0)$ degrees of freedom and the posterior of $\tau$ is $\text{Gamma}\{(n + \delta_0)/2, \bar{s}^2(n + \delta_0)/2\}$, where

$$\beta^* = \Lambda\mu_0 + (I - \Lambda)\hat{\beta},$$

$$\Lambda = (X'X + \Sigma_0^{-1})^{-1}\Sigma_0^{-1},$$

$$\hat{\beta} = (X'X)^{-1}X'Y,$$

$$\bar{s}^2 = (n + \delta_0)^{-1}[Y'(I - M)Y + (\hat{\beta} - \mu_0)'(\Lambda'X'X)(\hat{\beta} - \mu_0) + \gamma_0],$$

$$M = X'(X'X)^{-1}X.$$