from page 10

$$P(n_{AA}, n_{AB}, n_{BB} \mid n_A, n_B) = \frac{n!\ n_A!\ n_B!}{n_{AA}!\ n_{AB}!\ n_{BB}!\ (2n)!}\ 2^{n_{AB}}.$$

factorial computations are very time consuming.
But one can write

$$P(n_{AA}, n_{AB}, n_{BB} \mid n_A, n_B) = P(n_{AB} \mid n_A, n_B)$$

(because conditional on $n_A, n_B$ & $n$ there is only one free parameter)

$$P(n_{AB}+2 \mid n_A, n_B) = P(n_{AB} \mid n_A, n_B) \cdot \frac{4\, n_{AA}\, n_{BB}}{(n_{AB}+2)(n_{AB}+1)}$$

$$P(n_{AB}-2 \mid n_A, n_B) = P(n_{AB} \mid n_A, n_B) \cdot \frac{n_{AB}(n_{AB}-1)}{4(n_{AA}+1)(n_{BB}+1)}$$

- There are factorial calculations once you calculate $P(n_{AB} \mid n_A, n_B)$ for one value of $n_{AB}$.

- One even does not need to compute $P(n_{AB} \mid n_A, n_B)$

Why? Because one can ~~calculate~~ assume it is $= c$

then since $\sum P(n_{AB} \mid n_A, n_B) = 1$, then one can solve for $c$
$\forall\, n_{AB} \in \{$consistent with observation$\}$

$$P(n_{AB} = n_{AB} + 2) = \frac{n! \; n_A! \; n_B! \; 2^{n_{AB}} \cdot 2^{2}}{(n_{AA}-1)! \; (n_{AB}+2)! \; (n_{BB}-1)!}$$

$$= \left[ \frac{n! \; n_A! \; n_B! \; 2^{n_{AB}}}{(n_{AA})! \; (n_{AB})! \; (n_{BB})!} \right] * \frac{4 \; n_{BB} \; n_{AA}}{(n_{AB}+2)(n_{AB}+1)}$$

$$= p(n_{AB} = n_{AB}).$$

Probabilities

| $n_{AA}$ | $n_{AB}$ | $n_{BB}$ | |
|---|---|---|---|
| 0 | 10 | 7 | |
| 1 | 8 | 8 | |
| 2 | 6 | 9 | |
| 3 | 4 | 10 | → observed data |
| 4 | 2 | 11 | |
| 5 | 0 | 12 | |

$n_A = 10$

$n_B = 24$

$n = 34$

$$P(n_{AB}=6 \mid n_A, n_B) = c * \frac{4\, n_{AA}\, n_{BB}}{(n_{AB}+2)(n_{AB}+1)}$$

$$= c * \frac{4 * 10 \times 3}{(4+2)(4+1)} = 4c.$$

$$P(n_{AB}=8 \mid n_A, n_B) = P(n_{AB}=6 \mid n_A, n_B) \cdot \frac{4\, n_{AA}\, n_{BB}}{(n_{AB}+2)(n_{AB}+1)}$$

$$= 4c \times \frac{4 \times 2 \times 9}{(6+2)(6+1)}$$

$$= \frac{36}{7} c$$

$$P(n_{AB}=10 \mid n_A, n_B) = \frac{36}{7} c \times \frac{4 \times 1 \times 8}{(8+2)(8+1)} = \frac{64}{35} c$$

$$P(n_{AB}=2 \mid n_A, n_B) = c * \frac{n_{AB}(n_{AB}-1)}{4(n_{AA}+1)(n_{BB}+1)}$$

$$= c * \frac{4(4-1)}{4(3+1)(10+1)} = \frac{3c}{44}$$

$$P(n_{AB}=0 \mid n_A, n_B) = \frac{3}{44} c \cdot \frac{2(2-1)}{4(4+1)(11+1)}$$

$$= \frac{1}{1760} c$$

$$1 = c + 4c + \frac{36}{7}c + \frac{64}{35}c + \frac{3}{44}c + c \cdot \frac{1}{1760}$$

$$1 = c\left(1 + 4 + \frac{36}{7} + \frac{64}{35} + \frac{3}{44} + \frac{1}{1760}\right)$$

$$c = 0.0830523$$

$$P(n_{AB} = 4 \mid n_A, n_B) = \frac{n!\, n_A!\, n_B!\, 2^{n_{AB}}}{n_{AA}!\, n_{AB}!\, n_{BB}!\, (2n)!}$$

$$= \frac{17!\; 10!\; 24!\; 2^4}{3!\; 4!\; 10!\; (34)!}$$

$n_A = 10$

$n_B = 24$

$$= 0.0830523.$$

$n_A + n_B = 2n = 34$

$n = 17$