Using R Function `DynMed(···)` to Perform Bayesian Dynamic Mediation Analysis

Jing Huang[1] and Ying Yuan[2*]
[1]The Univeristy of Texas Health Science Center
[2]The University of Texas MD Anderson Cancer Center
* yyuan@mdanderson.org

1 Bayesian Dynamic Mediation Model

The Bayesian dynamic mediation model proposed by Huang and Yuan (2016) can be described as follows,

$$M_{it} = \gamma_i(t) + \alpha(t)X_{it} + \epsilon_{it} \tag{1}$$

$$Y_{it} = \delta_i(t) + \beta(t)M_{it} + \tau(t)X_{it} + \eta_{it} \tag{2}$$

$$\gamma_i(t) = \gamma(t) + u_{1i} \tag{3}$$

$$\delta_i(t) = \delta(t) + u_{2i}, \tag{4}$$

where at the first level of the model (i.e., equations (1) and (2)), $\alpha(t)$ is a function of time t, which quantifies how the relationship between the mediator variable M and the predictor variable X varies over time; and $\beta(t)$ is a function of t, which measures the variability over time of the relationship between the outcome variable Y and the mediator variable M, after adjusting for the predictor variable X. The subject-specific random processes $\gamma_i(t)$ and $\delta_i(t)$ are used to capture the potential heterogeneity across subjects. The vectors $\boldsymbol{\epsilon}_i = (\epsilon_{it_1}, \epsilon_{it_2}, \cdots, \epsilon_{it_{m_i}})$ and $\boldsymbol{\eta}_i = (\eta_{it_1}, \eta_{it_2}, \cdots, \eta_{it_{m_i}})$ represent random errors that are assumed to follow multivariate normal distributions with mean 0 and respective covariance matrices Σ_ϵ and Σ_η. At the second level of the model (i.e., equations (3) and (4)), $\gamma(t)$ and $\delta(t)$ specify population baseline processes, which are functions of time t; $\boldsymbol{u}_1 = (u_{11}, ..., u_{1i}..., u_{1n})$ and the vectors $\boldsymbol{u}_2 = (u_{21}, ..., u_{2i}..., u_{2n})$ represent random errors that are assumed to have multivariate normal distributions with mean 0 and respective covariance matrices Σ_γ and Σ_δ. Depending on the specific application, we can choose appropriate forms of first-level

covariance matrices Σ_ϵ and Σ_η and second-level covariance matrices Σ_γ and Σ_δ to obtain the desired correlation structures for Y and M. In `DynMed(···)`, we set $\Sigma_\epsilon = \sigma_\epsilon^2 I$, $\Sigma_\eta = \sigma_\eta^2 I$, $\Sigma_\gamma = \sigma_\gamma^2 I$ and $\Sigma_\delta = \sigma_\delta^2 I$, resulting in commonly used compound symmetric covariance matrices for Y and M.

The dynamic mediation effect is defined as $\alpha(t)\beta(t)$, which can vary over time. To avoid restrictive parametric assumptions, $\alpha(t)\beta(t)$ is modeled nonparametrically using penalized spline method, see Huang and Yuan (2016) for details.

2 Using `DynMed(···)`

2.1 `DynMed(···)`

Input arguments:

(1) `data` : the dataset to be analyzed. It should be organized in long format (each row contains the observations from a subject at a time point) and contains at least 5 columns:

- "id" : subject id
- "x" : predictor
- "m" : mediator
- "y" : outcome
- "t": measurement time standardized to $[0, 1]$, e.g., dividing the actual measurement time by the maximum measurement time in the data.

An example of data matrix:

```
      id      x      m       y  t
 [1,]  1  12.25 -27.36   30.62 0.07
 [2,]  1 -16.32  33.41  -40.63 0.13
 [3,]  1 -37.66  72.58  -72.29 0.20
 ...  ...  ...    ...     ...   ...
[13,]  1  -3.38  -4.27  -15.83 0.87
[14,]  1 -21.25 -45.66 -150.26 0.93
[15,]  1  33.79  82.00  274.71 1.00
[16,]  2 -37.00  80.74 -100.19 0.07
[17,]  2   0.37  -1.91    2.45 0.13
[18,]  2   1.98  -5.89    7.68 0.20
[19,]  2  -6.65  10.77   -7.88 0.27
[20,]  2  12.50 -18.33    6.29 0.33
 ...  ...  ...    ...     ...   ...
```

(2) `timepoint` : a vector of time points, at which the mediation effect is to be estimated. The time points should be standardized in the same way as the measurement time in the input data.

(3) `plot` : indicate whether or not to plot the estimated mediation effect curve. The default value is TRUE.

Outputs

- **Estimates of the mediation effect**: A matrix with 5 columns. The first column shows the time points specified by the augment `timepoint`; the second column shows the posterior (mean) estimate of the mediation effect at each of these specified time points; the third column shows the standard error of the estimated mediation effect; and the last two columns show the 95% credible intervals for the estimated mediation effects.

- **A plot of the estimated mediation effect curve** : If plot=TRUE, a plot of the estimated mediation effect curve will be generated.

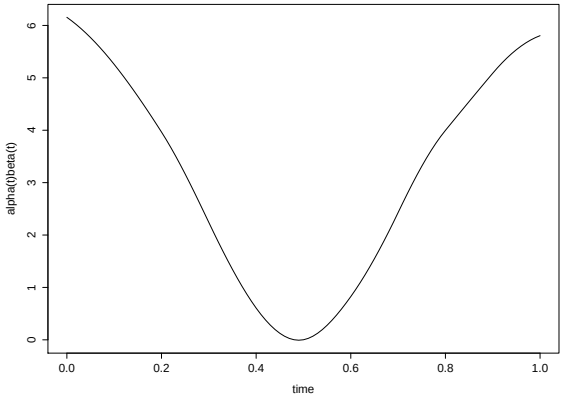
- **The posterior samples**: A matrix contains the MCMC samples of the mediation effect at the time points specified by the augment `timepoint`.

2.2 Example of using `DynMed(···)` to analyze data

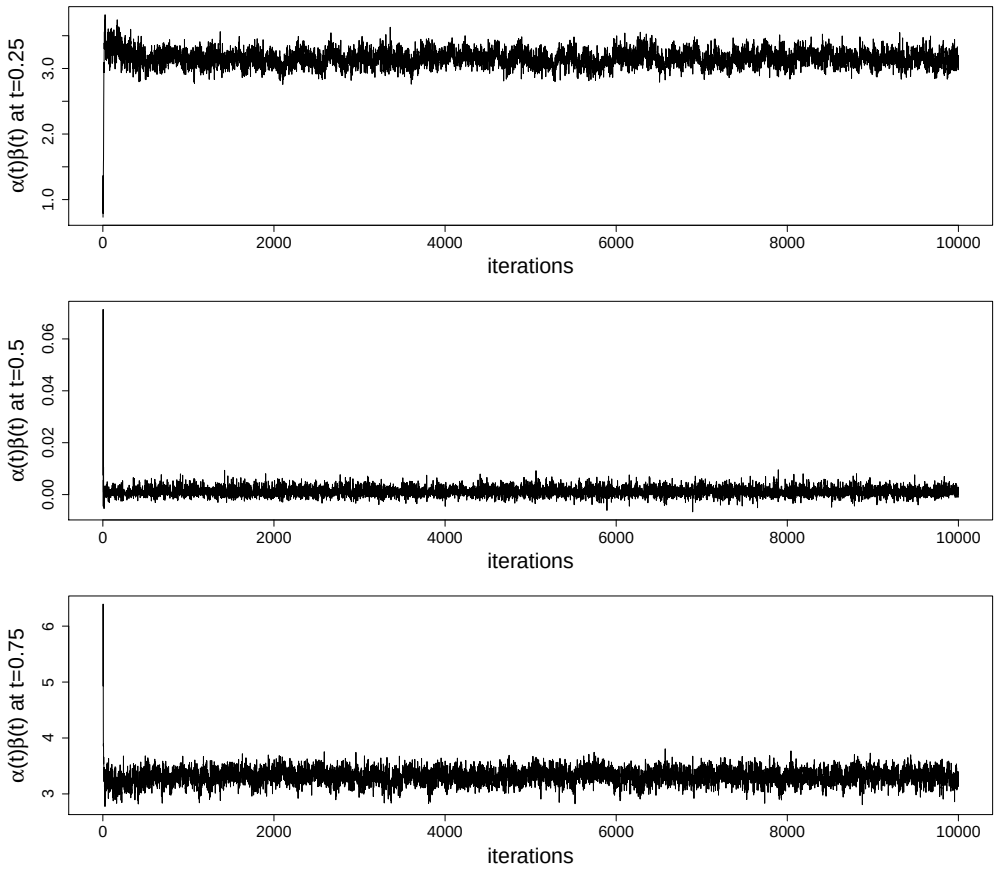
```
### read in R script
> source(‘‘directory where DynMed.R is saved/DynMed.R’’)

### read in dataset
> mydata = read.table(‘‘directory where the data is saved/exampledata.txt’’, header = T)

### perform Bayesian dynamic mediation analysis
### We are interested in estimating the mediation effect at standardized
###  time points 0.25, 0.5 and 0.75.
> dmresult = DynMed(data = mydata, timepoint = c(0.25,0.5,0.75), plot=T)
      t      mean         sd         95% CI      95% CI
[1,] 0.25 3.152026816 0.115408005  2.933191429 3.381097186
[2,] 0.50 0.001147799 0.001608855 -0.001781501 0.004694021
[3,] 0.75 3.318173722 0.130946522  3.050508497 3.569394015
```


alpha(t)beta(t)
0
1
2
3
4
5
6
0.0
0.2
0.4
0.6
0.8
1.0
time

```
### Generate the trace plot for MCMC samples
### Useful for convergence diagnosis for the MCMC
> par(mfrow=c(3,1))
> par(mar = c(4, 5, 2, 5), oma = c(3, 5, 3, 3),mgp=c(3, 1, 0))
> plot(c(1:10000), dmresult[1,],"l",
>      ylab = expression(paste(alpha,"(t)",beta,"(t)"," at t=0.25", sep="")),
>      xlab = "iterations",cex.lab=2,cex.axis=1.5)
> plot(c(1:10000), dmresult[2,],"l",
>      ylab = expression(paste(alpha,"(t)",beta,"(t)"," at t=0.5", sep="")),
>      xlab = "iterations",cex.lab=2,cex.axis=1.5)
> plot(c(1:10000), dmresult[3,],"l",
>      ylab = expression(paste(alpha,"(t)",beta,"(t)"," at t=0.75", sep="")),
>      xlab = "iterations",cex.lab=2,cex.axis=1.5)
```



3 Reference

Huang, J. and Yuan, Y. (2016) Bayesian Dynamic Mediation Analysis. *Psychological Methods*, to appear.