Homework assignment for Stat 630, Spring Semester, 2012. Answer each question precisely and completely. Be as brief as possible. Clearly define any notation before using it. Use clear, complete sentences when giving your answers and derivations. You may use any references you like, but all of your work must be your own. This homework assignment is due at the start of class, January 17.

1. Consider a screening test for a disease, denoted by “D” . Let “+” denote that the test was positive, indicating presence of the disease, with the complementary event of a negative test denoted by “-“ . In practice, the test is not always correct. Let Dc denote absence of the disease. Suppose that the test has “99% Sensitivity,” i.e. Pr(Test + | D) = .99, “95% Specificity,” i.e. Pr(Test – | Dc) = .95, and D has “.5% Prevalence,” i.e. Pr(D) = .005.
2. State Bayes’ Law, and use it to answer questions b and c.
3. If you just tested + , what is the probability that you actually have the disease?
4. If you test + a *second* time, assuming that the two tests are independent, what is the updated probability that you have the disease ?
5. Under a Bayesian model for a k-nary outcome Y taking on values in the set {1,2,…,k} with pj = Pr(Y=j) for j=1,…,k, so that p1 + . . . + pk = 1. Assume that **p** = (p1,..,pk-1) follows a Dirichelt(a1,…,ak) prior, where **a** = (a1,…,ak) are k known fixed positive-valued hyper-parameters. if a sample Y1,…,Yn  of size n is observed,
6. derive the joint distribution of **X** = (X1,…,Xk)given **p**, where Xj = I(Y1= j)+…+ I(Yn= j) = the number of observations out of n that fall in category j, for j=1,…,k.
7. derive the posterior distribution of [ **p | X** ]**.**
8. briefly explain these results in the case k=2, and give a simple example.
9. Suppose that Y follows a binomial distribution with parameters p and n, where 0 < p < 1 and n is a fixed positive integer. Suppose that p follows a beta(a,b) prior.
10. Derive the posterior distribution of [p | Y] . You may apply your answer to 2a, above.
11. Let Z be a future binomial variable, not yet observed, with parameters p and m, where m is a positive integer. Derive the predictive distribution of [Z | Y ].
12. Suppose that non-n egative integer-valued Y follows a Poisson distribution with mean  = 1/ (equivalently, rate ) and that  follows a gamma prior. Define “conjugate prior” and show that this prior is conjugate for the Poisson.
13. Suppose that Y1,…,Yn is a sample of n iid normally distributed real-valued random variables with mean  and variance 2.
14. Derive the distributions of the sample mean and the sample variance, and show that these two statistics are independent.
15. If  follows a normal prior, assuming known  =1, derive the posterior distribution of  given the sample mean.